

Becoming a Human Calculator

Parent Notes



Bringing out the *STAR* in kids!

Abra-Kid-Abra

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Thanks for having your child participate in our Becoming a Human Calculator program. These notes are provided to help you assist you child in mastering the tricks. The notes are based on our 15 hour camp. If your program is less hours, not all this material will be covered, so some of it will be extra. If you have questions on anything, please let us know. 314-961-6912 info@abrakid.com Good luck!

Folding a Paper in Half

Note: This is intended as an opening attention-grabber. It's interactive, gets them involved, and it's quick.

Effect: You ask students how many times they think they can fold a sheet of paper in half? I.e. the fold it in half, then in half again, then in half again, etc. Tell them that if anyone can fold it in half more than 9 times, you will give them a big prize. They try, and think they can do it, but they can't fold a paper in half more than, in most cases, 7x—certainly not more than 9x.

Props: Piece of paper for each student. (8.5x11 is good. Any size works.)

Secret/Teaching This: Why can't you fold a paper in half more than 9x? Draw a table like the 1st diagram below. Let the students tell you the answer for column 2. Diagram #2 has the answers when you keep doubling. Why can't you fold a paper in half 7, 8, or 9x? Because you are folding 128, 256, and 512 thicknesses of paper. When you try to fold more than 9x, you are folding over 1000 thicknesses of paper—which is virtually impossible!

Math Learning: Students learn to keep doubling numbers, and how quickly numbers add up when you keep doubling—much faster than you'd think!

After Fold #	# of thicknesses
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

After Fold #	# of thicknesses
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024

Lightning Addition Tricks

Materials: For each student: Lightning Addition/Subtraction/Division Worksheet, 4-6 notecards, and a pen or sharpee.

1. Two Step Addition

$$31+47=?$$

To make this problem easier to solve in your head, break it into 2 steps.

$$47=40 + 7.$$

$$31+40=71.$$

$$71+7=78.$$

Let's try another.

$$73+26=?$$

$$26=20+6.$$

$$73+20=93.$$

$$93+6=99.$$

Let's try the first section of practice problems on your addition/subtraction worksheet.

2. Step Count Method

This is another method of adding numbers.

$$14+23=?$$

Count in steps. $23=20+3$. Count first in steps of 10, then 3.

$$14-24-34-37.$$

Let's try another.

$$53+35=?$$

$$53-63-73-83-88.$$

Let's try your hand at the step method addition problems on your worksheet.

Then students make 2-3 flash cards for each of the above 2 problem types. Put a problem on 1 side, using 1 of the worksheet problems, or they can make up their own, and the answer on the back. Write big.

Human Calculator Addition Worksheet

Addition Part 1, Two Step

$16 + 23 = \underline{\quad\quad}$ $27 + 32 = \underline{\quad\quad}$ $51 + 47 = \underline{\quad\quad}$ $84 + 46 = \underline{\quad\quad}$ $165 + 32 = \underline{\quad\quad}$

$248 + 51 = \underline{\quad\quad}$ $456 + 25 = \underline{\quad\quad}$ $1026 + 71 = \underline{\quad\quad}$ $2514 + 63 = \underline{\quad\quad}$ $3548 + 35 = \underline{\quad\quad}$

Addition Part 2, Step Count Method

$13 + 25 = \underline{\quad\quad}$ $32 + 37 = \underline{\quad\quad}$ $41 + 46 = \underline{\quad\quad}$ $62 + 27 = \underline{\quad\quad}$ $112 + 26 = \underline{\quad\quad}$

$354 + 35 = \underline{\quad\quad}$ $527 + 48 = \underline{\quad\quad}$ $1250 + 33 = \underline{\quad\quad}$ $4517 + 22 = \underline{\quad\quad}$ $6849 + 34 = \underline{\quad\quad}$

ANSWERS - Human Calculator Addition Worksheet

Addition Part 1, Two Step

$16 + 23 = \underline{39}$

$27 + 32 = \underline{59}$

$51 + 47 = \underline{98}$

$84 + 46 = \underline{130}$

$165 + 32 = \underline{197}$

$248 + 51 = \underline{299}$

$456 + 25 = \underline{481}$

$1026 + 71 = \underline{1097}$

$2514 + 63 = \underline{2577}$

$3548 + 35 = \underline{3583}$

Addition Part 2, Step Count Method

$13 + 25 = \underline{38}$

$32 + 37 = \underline{69}$

$41 + 46 = \underline{87}$

$62 + 27 = \underline{89}$

$112 + 26 = \underline{138}$

$354 + 35 = \underline{389}$

$527 + 48 = \underline{575}$

$1250 + 33 = \underline{1283}$

$4517 + 22 = \underline{4539}$

$6849 + 34 = \underline{6883}$

Amazing Addition

Effect: You show a sheet with 5 rows of (primarily) 3-digit numbers. While you turn away, your assistant from the audience jots 5 numbers in a column, selecting a number from each column on your sheet. Someone with a calculator totals the five 3-digit numbers. They read off the 5 numbers to you. Within seconds, you total the five 3-digit numbers in your head, and correctly announce the total!

Props: For each child: Amazing Addition Sheet (8.5x11).
For the show: 11x17 laminated version.

Secret: As the spectator is reading off the numbers, you listen only to the last digits and add them in your head. Suppose, e.g. that the sum is 20. That's the last part of the answer. To get the first part, subtract the sum from 50—in this example, $50-20=30$, and that's the first part of the answer. Put them together to get the answer: 3020! If the sum is 24, the total would be 2624.

A	B	C	D	E
366	345	186	872	756
69	840	582	971	558
168	246	87	575	657
762	147	285	377	954
960	543	483	179	855
564	48	780	674	459

Presentation:

“I’m going to attempt a very dangerous mathematical trick. Many magicians have perished attempting this. May I have a volunteer from the audience please. (Introduce yourself.) I have a bunch of varied numbers on this sheet. Most are 3 digits. A few are 2-digits. I’m going to look away, and I’d like you to write 5 numbers in a column, choosing 1 number from each column. For instance, from column A you might choose 366; from column B, 147, etc. Pick 1 number from each column. Write your 5 numbers in a column so they can be added up. Let me know when you are finished. OK? (While spectator is jotting his numbers, talk a little to fill the dead air.) While he is choosing his numbers, is there anyone in the audience with a calculator—on their phone, perhaps, or otherwise? (yes) Great. As he writes the numbers, would you add them up please. Hopefully he won’t choose such high numbers that they won’t fit on your calculator.

Have you written 5 numbers in a column? (yes) Great. And do you have the total? (yes) Very well. I’m going to ask you to read the 5 numbers to me and I will attempt to add them in my head. Ready? What numbers did you choose? (As he names them, you add the last digits of each in your head. Suppose the total is 27. You announce the total.) Is the total 2327? (yes) A round of applause for my assistants!”

Notes: Why does this trick work? Do you notice anything interesting about the numbers in column A? Answer: They all have the same middle number. And their first & last digits all add to 9. What about the other columns? Same. Their middle numbers in each column are the same & their first & last digits all add to the same number. The middle digits are: 4,5,6,7, & 8. The 5 numbers selected will always have these 5 numbers as their middle digits. What do they total? 30. The first & last numbers add to: 13, 10, 7, 8, & 9. What do those add to? 47. $47+3$ (from the 30) = 50. This points you toward the explanation.

Doing the Dishes

Comments: This is a gag, not a show trick. It illustrates the power of geometric progression.

Effect: You offer to do the dishes nightly and forgo your allowance for 1 cent the first night, 2 cents the second, 4 cents the third, etc. for 30 nights. When you parents say ok, you surprise them with a huge bill!

Props: Dishes Card with contract on 1 side and bill on the other.

Secret: Doubling adds up very quickly—quicker than you might think!

Presentation:

“I have a trick for you to play on your parents. Would you like to play a trick on your parents? (yes!)

Go home tonight and tell them that you have a proposition for them. You would like to start doing the dishes every night after dinner. How many of you like doing the dishes? Not only that, but you want to do the dishes so badly, that you’ll even forgo your allowance! All you ask in return is that they pay you 1 cent the first night, 2 cents the second, 4 cents the third night, doubling to 8 cents the fourth, as so on, for 30 days. How many of you would be interested in doing that? (hopefully few)

How much would the money you charge add up to in 30 days—what do you think?” (Ask for some guesses.) Let’s add it up. (Draw 2 columns: “Day” and “Fee”. Under day, put 1,2,3,4,... Under fee, have the kids help you for a number of days until it becomes very difficult. Then, pass out the bills and show that the total is, believe it or not, over \$10 million!)

Pass out the contracts. Tell them that if they get their parents to sign, it’s not a real contract, so they can’t really collect the money. And they should tell the parents that if they get worried. But they can have some fun with this!

Math Learning: Compounding—doubling—adds to a huge sum quickly. You might even mention that this is why it is important to save money. It earns interest, which earns interest, which earns more interest. After 20-30 years, a small amount saved adds up to a large amount earned.

Contract	
I, _____,	parent’s name
agree to pay _____	your name
1 cent for doing the dishes the first night, 2 cents for the second night, 4 cents for the third night, double that to 8 cents for the fourth night, and so on for the next 30 days.	
_____ Parent’s signature	
<i>This agreement is just for fun.</i>	

Day	Pay	Day	Pay
1.	\$.01	16.	\$327.68
2.	\$.02	17.	\$655.36
3.	\$.04	18.	\$1310.72
4.	\$.08	19.	\$2621.44
5.	\$.16	20.	\$5242.88
6.	\$.32	21.	\$10485.76
7.	\$.64	22.	\$20971.52
8.	\$1.28	23.	\$41943.04
9.	\$2.56	24.	\$83866.08
10.	\$5.12	25.	\$167772.16
11.	\$10.24	26.	\$335544.32
12.	\$20.48	27.	\$671088.64
13.	\$40.96	28.	\$1342177.28
14.	\$81.92	29.	\$2684354.56
15.	\$163.84	30.	\$5368709.12
		Total	\$10737418.23

Lightning Multiplication

Effect: The audience holds up flash cards with a variety of 2-digit x 2-digit multiplication problems. You lightning calculate them in your head, announcing the correct answers!

Props: For each student: Multiplication Worksheet, 4-6 3x5 note cards, & pens or sharpees.

How You Do It: There are 4 different types of problems you lightning calculate:

1st Type of Problem: Squaring 2-digit #s ending in 0. $_0 \times _0 = ?$

E.g. $30 \times 30 = ?$ $3 \times 3 = 9$. Tack on 2 0's to get 900. So $30 \times 30 = 900$.

$60 \times 60 = ?$ $6 \times 6 = 36$. Tack on 2 0's. $60 \times 60 = 3600$. Just squaring #s 1-9 and appending 2 0's. Easy!

2nd Type of Problem: Squaring 2-digit #s ending in 5. $_5 \times _5 = ?$

What is 25×25 ? 625. How do you know? A) 25 is between 20 and 30. Multiply $2 \times 3 = 6$. That's the first part. B) Multiply the 2 last digits together. $5 \times 5 = 25$. That's the last part. 625.

How about $45 \times 45 = ?$ 45 is between 40 & 50. So multiply $4 \times 5 = 20$. That's the first part. $5 \times 5 = 25$. That's the 2nd part. Put them together: 2025. (Note: 4×5 is really $40 \times 50 = 2000$, $+ 25 = 2025$)

Activity: Complete 2 rows of the 2-digit Multiplication Worksheet to hone your skills.

3rd Type of Problem: Multiplying, e.g. 32×38 . 2-digit #s with the same first digit, whose last digits add to 10.

What is 32×38 ? Use the same approach. $3 \times 4 = 12$. $2 \times 8 = 16$. Put them together: 1216.

$73 \times 77 = ?$ $7 \times 8 = 56$. $3 \times 7 = 21$. 5621.

$51 \times 59 = ?$ $5 \times 6 = 30$. $1 \times 9 = 9$. 309? No, 3009. If multiply the 2nd digits & get a 1 digit number, e.g. 9, make it 09.

Complete Part 2 of the worksheet to hone your skills.

4th Type of Problem: $25 \times 45 = ?$ Last digit is 5. The 2 #s are 20, 40, 60, 80, or 100 apart.

What's in the middle? 35. $35 \times 35 = 1225$. 45 is 10 away from 35, and 25 is 10 away from 35. $10 \times 10 = 100$. Subtract 100 from 1225 and you have the answer: 1125!

$55 \times 75 = ?$ 65 is midpoint. $65^2 = 4225$. 55 & 75 are each 10 away from 65. $10^2 = 100$. Answer: $4225 - 100 = 4125$.

One more. $35 \times 75 = ?$ Midpoint: 55. $55^2 = 3025$. 35 & 75 are each 20 away. $20^2 = 400$. Answer: $3025 - 400 = 2625$.

Why does this work?

$$25 \times 25 = 625$$

$$24 \times 26 = 624$$

$$23 \times 27 = 621$$

$$22 \times 28 = 616$$

$$21 \times 29 = 609$$

$$20 \times 30 = 600$$

How many is each answer from 625? $24 \times 26 = 1$ less. $23 \times 27 = 4$ less. $22 \times 28 = 9$ less. $21 \times 29 = 16$ less. $20 \times 30 = 25$ less. What are these numbers? Squares of 1, 2, 3, 4, & 5. So: $23 \times 27 = ?$ We know $25 \times 25 = 625$. 23 & 27 are each 2 away from 25. $2^2 = 4$. $625 - 4 = 621$. This is another way of solving this sort of problem.

In 25×45 , $35 \times 35 = 1225$. 25 & 45 are each 10 away. $10^2 = 100$. Answer is 100 less than 1225, or 1125.

Presentation:

1. You have a packet of 2-digit multiplication flash cards. There are several different colors of cards, each corresponding to a different type of problem above. 1 side has the problem, which can be shown to the student. The other side contains the answer.
2. To practice, have the kids stand shoulder to shoulder on stage. Randomly hold up cards, reading the problem. Each child gets read a different card and tries to lightning calculate the answer. They can write it on the board if they need to.
3. In the show:
 - a. The lightning calculators stand shoulder to shoulder on stage.
 - b. Explain that we have here a series of math wizards. They are going to demonstrate their mathematical prowess by trying to lightning calculate in their heads the answer to a series of 2-digit multiplication problems.
 - c. Show that you have a stack of cards with a variety of multiplication problems on them. One side has the problem, the other side, the answer. Have members of the audience select approximately 20 cards (4 or so from each of the 5 groups).
 - d. One at a time, audience members will hold up their card, and read out loud their math problem. The next lightning calculator whose turn it is will try to lightning calculate it.
 - e. Depending on how many students you have, each should get ~2-4 turns.

Notes: Practice at home using the Multiplication Problems sheet.

Teaching Note: The 4th Type of Problem above is designed to stretch what they've learned. This is a great accomplishment if you can get kids to solve these type problems in their heads.

Students Making Their Own Flash cards: Have students continue to build their lightning calc flashcard decks by writing, for each of the above problem types, 2-3 flash cards. Put a problem on 1 side, using 1 of the worksheet problems, or they can make up their own, and the answer on the back. Write large.

Human Calculator Two Digit Multiplication Worksheet

Part 1, Squaring Tens

$20^2 = \underline{\quad}$ $30^2 = \underline{\quad}$ $40^2 = \underline{\quad}$ $50^2 = \underline{\quad}$ $60^2 = \underline{\quad}$ $70^2 = \underline{\quad}$ $80^2 = \underline{\quad}$ $90^2 = \underline{\quad}$

Part 2, Squaring _5's:

$15 \times 15 = \underline{\quad}$ $25 \times 25 = \underline{\quad}$ $35 \times 35 = \underline{\quad}$ $45 \times 45 = \underline{\quad}$ $55 \times 55 = \underline{\quad}$

$65 \times 65 = \underline{\quad}$ $75 \times 75 = \underline{\quad}$ $85 \times 85 = \underline{\quad}$ $95 \times 95 = \underline{\quad}$

Part 3, First Digits-Same, Last Digits Add to Ten:

$34 \times 36 = \underline{\quad}$ $33 \times 37 = \underline{\quad}$ $32 \times 38 = \underline{\quad}$ $31 \times 39 = \underline{\quad}$ $28 \times 22 = \underline{\quad}$

$41 \times 49 = \underline{\quad}$ $46 \times 44 = \underline{\quad}$ $58 \times 52 = \underline{\quad}$ $53 \times 57 = \underline{\quad}$ $67 \times 63 = \underline{\quad}$

$73 \times 77 = \underline{\quad}$ $88 \times 82 = \underline{\quad}$ $84 \times 86 = \underline{\quad}$ $93 \times 97 = \underline{\quad}$ $99 \times 91 = \underline{\quad}$

Part 4, _5s - 20 or 40 apart

$25 \times 45 = \underline{\quad}$ $35 \times 55 = \underline{\quad}$ $55 \times 75 = \underline{\quad}$ $75 \times 95 = \underline{\quad}$ $85 \times 105 = \underline{\quad}$

$15 \times 55 = \underline{\quad}$ $25 \times 65 = \underline{\quad}$ $45 \times 85 = \underline{\quad}$ $55 \times 95 = \underline{\quad}$ $35 \times 75 = \underline{\quad}$



ANSWERS--Human Calculator Two Digit Multiplication Worksheet

Part 1, Squaring Tens

$$20^2 = 400 \quad 30^2 = 900 \quad 40^2 = 1600 \quad 50^2 = 2500 \quad 60^2 = 3600 \quad 70^2 = 4900 \quad 80^2 = 6400 \quad 90^2 = 8100$$

Part 2, Squaring _5's:

$$15 \times 15 = 225 \quad 25 \times 25 = 625 \quad 35 \times 35 = 1225 \quad 45 \times 45 = 2025 \quad 55 \times 55 = 3025$$

$$65 \times 65 = 4225 \quad 75 \times 75 = 5625 \quad 85 \times 85 = 7225 \quad 95 \times 95 = 9025$$

Part 3, First Digits-Same, Last Digits Add to Ten:

$$34 \times 36 = 1224 \quad 33 \times 37 = 1221 \quad 32 \times 38 = 1216 \quad 31 \times 39 = 1209 \quad 28 \times 22 = 616$$

$$41 \times 49 = 2009 \quad 46 \times 44 = 2024 \quad 58 \times 52 = 3016 \quad 53 \times 57 = 3021 \quad 67 \times 63 = 4221$$

$$73 \times 77 = 5621 \quad 88 \times 82 = 7216 \quad 84 \times 86 = 7224 \quad 93 \times 97 = 9021 \quad 99 \times 91 = 9009$$

Part 4, _5s - 20 or 40 apart

$$25 \times 45 = 1125 \quad 35 \times 55 = 1925 \quad 55 \times 75 = 4125 \quad 75 \times 95 = 7125 \quad 85 \times 105 = 8925$$

$$15 \times 55 = 825 \quad 25 \times 65 = 1625 \quad 45 \times 85 = 3825 \quad 55 \times 95 = 5225 \quad 35 \times 75 = 2625$$

Card Prediction (15♦)

Effect: You place a prediction card face down on the table. Spectator selects a card. You ask him to multiply it by 3. For instance, if he picked the 2♥, $x3=6♥$. If he chose the 3♣, $x3 = 9♣$. Whatever his card is, he multiplies it by 3, and that'll be your prediction! Spectator says that is not possible. Why, you ask. Because, says the spectator, you'd have to have the 15♦! You turn over your card. Sure enough, it is the 15♦!

Props: 15♦ card & a deck of cards.

Secret/Preparation: While the spectator thinks he has a free selection, you actually force him to choose the 5♦. Before you start, place the 5♦ so it is the 10th card from the top.

Presentation: Have the 15♦ face down on the table, off to the side. Invite a spectator up from the audience. Have him come around to the same side of the table as you, facing the audience. Ask him to name any number from 10-20, then deal (from the top of the deck) that # of cards into a (face down) pile.

Ask spectator to pick up the pile he just dealt, add the digits of his number, & deal that many. E.g. if spectator picks 15, he deals 15 in a pile. Then from that pile, he deals 6 cards (1+5) onto the table. The last card (the 6th one in this example) is his card—have him set it aside on the table, as you reassemble the rest of the deck. Pick up the 15♦ (don't let them see its face, of course). "I have taken the liberty of making a prediction. This (hold up 15♦, back facing audience & spectator) is my prediction card. Whatever card you picked, multiply it by 3 & that will be my card. For example, if you have the 2♥, mine will be the 6♥. If you picked the 3♣, mine will be the 9♣. Would you look at your card and tell me—when you multiply it by 3, what do you get? (15♦) This is no time for jokes. I am trying to conduct a performance. The card that you're holding, what do you get when you triple it? (15) May we see the card please. (He shows it's the 5♦. Pick up the deck and spread through the cards face up, looking through it.) I must have given you the wrong card. Shall we start over from the beginning?

Would you turn over the card, let's see how close I was. (Hold up the 15♦ so everyone can see it.) Thank you. Let's give our assistant a round of applause.

Tips:

- Act like you messed up. Then you show that you really did get it right.

Math Teaching:

In the patter, give 2-3 examples of multiplying the value by 3, as above. This helps teach multiplying by 3.

You can also teach why the force works, which is as follows.

Spectator deals any number of cards from 10-20.

10	$1+0=?$ (1)	$10-1=?$ (9)
11	$1+1=?$ (2)	$11-2=?$ (9)
15	$1+5=?$ (6)	$15-6=?$ (9)

The point: Many different equations = the same number. No matter how many cards the spectator deals, it always = your card!

Question: These equations = 9. Why is it the 10th card, not the 9th? Answer: When you deal the 2nd pile, you start counting with the first card, which, in effect, gets double counted. If you dealt 10 cards on the table in a row, then started on the last card and counted 1 as the next card, it would be 9th from the top.

Math Quiz (Comical)

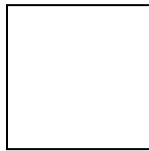
Effect: This is a gag. You explain that you took a math test the other day. You had no idea of the answers. Yet you get all the answers right!

Props: Math Quiz sheets—4 per person. This gives each a few to use. When you write on one, it's hard to use again, unless you write in pencil, then erase it so it can be re-used. You might suggest this to the kids.

Blank Math Quiz

Math quiz

Part One: Geometry
Divide this box into 4 equal parts.



Part Two: Multiplication

$2 \times 9 = \underline{\quad}$

$3 \times 9 = \underline{\quad}$

$4 \times 9 = \underline{\quad}$

$5 \times 9 = \underline{\quad}$

$6 \times 9 = \underline{\quad}$

$7 \times 9 = \underline{\quad}$

$8 \times 9 = \underline{\quad}$

$9 \times 9 = \underline{\quad}$

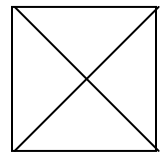


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Filled In Math Quiz

Math Quiz

Part One: Geometry
Divide this box into 4 equal parts.



Part Two: Multiplication

$2 \times 9 = \underline{18}$

$3 \times 9 = \underline{27}$

$4 \times 9 = \underline{36}$

$5 \times 9 = \underline{45}$

$6 \times 9 = \underline{54}$

$7 \times 9 = \underline{63}$

$8 \times 9 = \underline{72}$

$9 \times 9 = \underline{81}$



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Presentation: “I took a math quiz recently. I looked at the first question—geometry. I don’t know geometry, so I skipped it. I looked at part 2, multiplication. I didn’t know those answers either. I counted to see how many I’ll miss. (Start in the top blank & work down. Write 1, 2,3, 4...thru 8 in the bottom blank.) 8. Oh boy. Let me double check that to make sure I’ll miss that many. (Now count 1-8 again, this time going up from the bottom. 1,2,3,4,...8. When you are done, you have put the correct answers in each blank—18 in the top one, 27 in the next, 36 in the next,... 81 in the bottom blank.)

The geometry question, I don’t know that either, so I’ll cross it out. (Fill the box with an X.) I didn’t know any of the answers. I turned in my test. Imagine my surprise when I got it back—100%! It pays to be a magician!

Math Learning: 1) Multiplying by 9’s. 2) geometry—dividing a square into 4 equal parts.

Lightning Multiplication Tricks #2

Effect: You show a set of flash cards with challenging arithmetic problems. Several members of the audience each pick 1. One by one they hold up their cards. You lightning calculate in your head and say the answers correctly! The audience verifies you are correct, as the answers are on the back of the card.

Props: For each student: Multiplication Worksheet, 4-6 3x5 note cards, & pens or sharpies.

Trick #1: Multiplying by 9

9×25 , for instance, is a tough problem. Multiply 25 by 10: $10 \times 25 = 250$. Then take 1 of the 25's away. $250 - 25 = 225$.

$9 \times 22 = ?$ $10 \times 22 = 220$. $220 - 22 = ?$ $220 - 20 = 200$. Minus 2 more = 198.

Got it? Do the first row of problems on your worksheet.

Trick #2: Multiplying x 19, x 29, x 39,...

The same is true when multiplying x 19 or 29 or 39,...

For example, $19 \times 3 = ?$ Seems tough. Round the 19 to 20 to make it easier. $20 \times 3 = 60$. Minus 1 of the 3's = 57. $29 \times 5 = ?$ $30 \times 5 = 150$. $-5 = 145$.

Try the 2nd set of problems on your worksheet.

Trick #3: $101 \times __ = ?$

$101 \times 67 = ?$ 6767. The number (67) repeated!

Why does this work? $100 \times 67 = ?$ 6700. Plus 1 more 67 = 6767.

$101 \times 79 = ?$ 7979

Trick #4: $1001 \times ___ = ?$

$1001 \times 432 = ?$ 432432.

Similar to 101x. $432 \times 1000 = 432000$. + 1 more 432 = 432432.

Try the problems for 101x and 1001x in your worksheet.

Trick #5: $99 \times __ = ?$

$99 \times 84 = ?$

$100 \times 84 = 8400$. Since it's 99×84 , it's going to be a little less. In fact, it'll be 83 $__$. $8400 - 84$ will be 83 $__$.

So here's the trick: $99 \times 84 = ?$ A) 1 less than the number is the first part. 83 $__$. B) to get the 2nd part, put the 9s compliments of the first 2 numbers. $8 + __ = 9$? 1. $3 + __ = 9$? 6. 16. So the answer is 8316.

$99 \times 43 = ?$ 42 is the first part. 9s compliment to 4 & 2: 57. Answer: 4257.

Try these problems in your worksheet.

Presentation: If you are the only math wiz in the room, have several cards selected and lightning calculate the answers. If there are several math whizzes, they stand shoulder to shoulder. The audience selects several flash cards and holds them up 1 at a time. The math whizzes go down the line, each correctly answering.

Students Making Their Own Flash cards: Have students continue to build their lightning calc flashcard decks by writing, for each of the above problem types, 2-3 flash cards. Put a problem on 1 side, using 1 of the worksheet problems, or they can make up their own, and the answer on the back. Write large.

Human Calculator Lightning Multiplication II Worksheet

Part 1: Multiplying by 9

$9 \times 25 = \underline{\quad}$ $9 \times 50 = \underline{\quad}$ $9 \times 12 = \underline{\quad}$ $9 \times 15 = \underline{\quad}$ $9 \times 35 = \underline{\quad}$ $9 \times 45 = \underline{\quad}$ $9 \times 75 = \underline{\quad}$

Part 2: Multiplying by 19, 29, 39,...

$19 \times 4 = \underline{\quad}$ $19 \times 5 = \underline{\quad}$ $29 \times 2 = \underline{\quad}$ $3 \times 29 = \underline{\quad}$ $39 \times 3 = \underline{\quad}$

$49 \times 2 = \underline{\quad}$ $49 \times 4 = \underline{\quad}$ $59 \times 2 = \underline{\quad}$ $69 \times 5 = \underline{\quad}$ $89 \times 5 = \underline{\quad}$

Part 3 & 4: $101 \times \underline{\quad} = ?$ $1001 \times \underline{\quad} = ?$

$101 \times 36 = \underline{\quad}$ $101 \times 57 = \underline{\quad}$ $87 \times 101 = \underline{\quad}$ $273 \times 1001 = \underline{\quad}$ $1001 \times 391 = \underline{\quad}$

Part 5: $99 \times \underline{\quad} = ?$

$99 \times 23 = \underline{\quad}$ $99 \times 36 = \underline{\quad}$ $99 \times 48 = \underline{\quad}$ $75 \times 99 = \underline{\quad}$ $97 \times 99 = \underline{\quad}$

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ANSWERS--Human Calculator Lightning Multiplication Worksheet

Part 1: Multiplying by 9

$$9 \times 25 = 225 \quad 9 \times 50 = 450 \quad 9 \times 12 = 108 \quad 9 \times 15 = 135 \quad 9 \times 35 = 315 \quad 9 \times 45 = 405 \quad 9 \times 75 = 675$$

Part 2: Multiplying by 19, 29, 39,...

$$19 \times 4 = 76 \quad 19 \times 5 = 95 \quad 29 \times 2 = 58 \quad 3 \times 29 = 87 \quad 39 \times 3 = 117$$

$$49 \times 2 = 98 \quad 49 \times 4 = 196 \quad 59 \times 2 = 118 \quad 69 \times 5 = 345 \quad 89 \times 5 = 445$$

Part 3 & 4: $101 \times _ _ = ?$ $1001 \times _ _ _ = ?$

$$101 \times 36 = 3636 \quad 101 \times 57 = 5757 \quad 87 \times 101 = 8787 \quad 273 \times 1001 = 273,273 \quad 1001 \times 391 = 391,391$$

Part 5: $99 \times _ _ = ?$

$$99 \times 23 = 2277 \quad 99 \times 36 = 3564 \quad 99 \times 48 = 4752 \quad 75 \times 99 = 7425 \quad 97 \times 99 = 9609$$

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Mind Blowing Multiplication Stunt

Effect: You do two 2-digit x 2-digit multiplication problems in your head, using numbers called out by the spectator, and correctly add the totals.

E.g. Spectator calls out a 2-digit number--e.g. 47. You are feeling lucky, so you write it 2x. (Below) Spectator calls out another 2-digit number. E.g. 36. You write this. To make things a little harder, you write another number--63. Looking away, you say you'll multiply both problems in your head & add the products. You jot your answer on a sheet of paper, as spectator computes, using a calculator. Spectator gives answer. You turn your answer around. It's right!

$\begin{array}{r} 47 \\ \times 36 \\ \hline 1692 \end{array}$	$\begin{array}{r} 47 \\ \times 63 \\ \hline 2961 \end{array}$
$1692 + 2961 = 4653$	

Props: Blackboard or sheet of paper with backing (e.g. book or notepad), marker, & calculator

Secret: 1) The 2nd # that you seem to randomly come up with in the 2nd problem is the 9 compliment of the 2nd # in the 1st problem. I.e. 36 was called out by the spectator in this example. What # added to 36 will = 99? 63. 2) To get the total, use the first # called out by the spectator, in this case, 47. Write 1 less, i.e. 46. Then append the 9 compliment, in this case, 53. So your answer is 4653.

Presentation:

"I'd like to try something with multiplication. You, sir, can you name any 2-digit number please. (Suppose 28.) I will write 28 on the board. In fact, I'm feeling lucky, so I'm going to write it 2x. (Do so.)

Another person, can you please name any other 2-digit number. (57. Jot it below the first 28.) Hmm. That's pretty hard. I better make the 2nd equation a little easier. (Jot the 9 compliment of 57 below the 2nd 28--i.e. 42. Look away, remembering the first number, 28.)

I am going to try to multiply both problems in my head. Then add the products together and give you the total. I may get a headache doing all this mental arithmetic, but it's worth it for your entertainment! I'll jot down what I come up with. Meanwhile, here is a calculator. Would you multiply each problem, add the answers, and let me know when you have a total. (Jot 1 less than 28, 27. Then the 9s compliment, 72. So 2772. Spectator says he has the total.) What did you come up with? (2772) Sometimes I don't get this right. (Pause) Fortunately, though, in this case, I did! (Turn your sheet around to show 2772!) How about a round of applause for my assistant!"

Notes:

- If spectator's 2nd 2-digit number is above 50, comment that that is a tough equation, you'd better make the 2nd one a little easier! If the number is below 50, comment that you'll make the 2nd one a little harder. It should appear that your 2nd number is chosen haphazardly, at random.
- Why does this work? Suppose the 1st number is 28. You are, in effect, multiplying it by 99. If you multiplied it by 100, what would you get? Tack on 2 zeros: 2800. But you're multiplying it x 99, so it's 2800 - 28. The answer will be 27___. I.e. 1 less than 28 for the 1st 2 digits. Last 2 digits are 72. This is the same as taking the 9s compliment of the 1st 2 digits.

Missing Digit Follow Up

This is a great follow-up to the Mind Blowing Multiplication Stunt.

Effect: You have a 4 digit total from the Mind Blowing Multiplication stunt. Turn away, and ask the spectator to multiply the total by any 1 digit number, and write the product on the board. Ask spectator to circle any digit (not a zero). Ask what number he multiplied it by? (e.g. 5) Say that you will multiply this in your head. Have spectator read off the remaining digits of the answer. You reveal the circled number—i.e. the missing digit!

$$\begin{array}{r} 4653 \\ \times 5 \\ \hline 23265 \end{array}$$

Props: Blackboard or sheet of paper with backing (e.g. book or notepad), marker, & calculator

Secret: As spectator reads the numbers (usually there are 4), add them in your head. Then add the digits until you get down to a single number. Subtract that number from 9, and you'll have the missing digit! E.g. here, $2+3+2+6=13$. $1+3=4$. $9-4=5$, which is the missing digit!

Presentation:

“Since we’re on a roll, let’s try something else. You have a total. Multiply it by any 1 digit number, and write the answer on the board. I won’t look. Let me know when you are done. (Spectator multiplies $4653 \times 5 = 23265$ and writes this on board, and says she’s done.) What number did you multiply it by? (5) Alright, I’m going to multiply this in my head (concentrate). Got it. In the answer you got, would you circle any number, not a zero. Have you done that? (yes) OK. Would you tell me the other numbers in your answer? (2326) Was the last number in your answer that you circled...5?”

Notes:

- Why does this work? In any number that is a multiple of 9, the digits add to 9. The original number in this trick is $99 \times$ something. 99 is a multiple of 9. Therefore, the product is a multiple of 9. When you multiply a multiple of 9 by any number, the product is also a multiple of 9. So the product is a multiple of 9. The digits add to 9. You add the digits to 1 number, subtract from 9, and you have it.
- Variation: You can also add the digits and instead of adding them together & subtracting from 9, you could subtract them from the next highest multiple of 9. Same result.
- This sequence of tricks—Mind Blowing Multiplication Stunt & Missing Digit Follow Up—is very powerful!
- You can multiply the total by a 2-digit number and it still works, though you have an extra digit to deal with.

Rubber Pencil Gag

Effect: Hand someone a pencil to jot something down, but they can't. It's rubber!

Props: Rubber pencil

Can you come up with ideas of how you can have fun with this item?



Subtraction Quickies

Challenge #1: If you take 12 apples from 17 apples, how many do you have?

Answer: 12.

Challenge #2: Sarah had 5 apples and ate all but 3. How many did she have left?

Answer: 3.

Challenge #3: If you have \$2 in 1 pants pocket, \$3 in a 2nd pants pocket, and \$5 in a 3rd pants pocket, what do you have?

Answer: Somebody else's pants

Lightning Subtraction

Materials: For each student: Lightning Add/Subtract/Divide Worksheet, 4-6 3x5 note cards, & pens or sharpies.

1. Two-Step

60-27 seems like a tough problem to solve in your head.
However, if you take it in 2 steps, it's more manageable.
Break 27 into 20 and 7.
 $60-20=40$.
 $40-7=33$. Quick & easy.

Let's try another.
 $140-65=?$
 $60=60+5$.
 $140-60=80$.
 $80-5=75$.

Try the problems on the worksheet.

2. Overshooting

This technique is good when you are subtracting a number that is close to 100, or a multiple of 100.

$170-95=?$

$95=100-5$. We'll subtract 100, which is easy. Then add back 5, which is equivalent to subtracting 95.
 $170-100=70$.
 $70+5=75$, which is the answer.

Let's try another.

$221-87=?$
 $221-100=121$.
 $121+13=134$.

Try the worksheet problems.

Students Making Their Own Flash cards: Have students continue to build their lightning calc flashcard decks by writing, for each of the above problem types, 2-3 flash cards. Put a problem on 1 side, using 1 of the worksheet problems, or they can make up their own, and the answer on the back. Write large.

Human Calculator Subtraction Worksheet

Part 1, Two Step

$40 - 13 = \underline{\quad\quad}$ $70 - 21 = \underline{\quad\quad}$ $80 - 52 = \underline{\quad\quad}$ $130 - 44 = \underline{\quad\quad}$ $190 - 76 = \underline{\quad\quad}$

$56 - 24 = \underline{\quad\quad}$ $87 - 36 = \underline{\quad\quad}$ $96 - 65 = \underline{\quad\quad}$ $124 - 31 = \underline{\quad\quad}$ $347 - 64 = \underline{\quad\quad}$

Part 2, Overshooting

$160 - 85 = \underline{\quad\quad}$ $473 - 86 = \underline{\quad\quad}$ $131 - 88 = \underline{\quad\quad}$ $342 - 92 = \underline{\quad\quad}$ $285 - 96 = \underline{\quad\quad}$

$514 - 185 = \underline{\quad\quad}$ $620 - 191 = \underline{\quad\quad}$ $708 - 289 = \underline{\quad\quad}$ $856 - 298 = \underline{\quad\quad}$ $997 - 398 = \underline{\quad\quad}$

ANSWERS - Human Calculator Subtraction Worksheet

Part 1, Two Step

$40 - 13 = \underline{27}$

$70 - 21 = \underline{49}$

$80 - 52 = \underline{28}$

$130 - 44 = \underline{86}$

$190 - 76 = \underline{114}$

$56 - 24 = \underline{32}$

$87 - 36 = \underline{51}$

$96 - 65 = \underline{31}$

$124 - 31 = \underline{93}$

$347 - 64 = \underline{283}$

Part 2, Overshooting

$160 - 85 = \underline{75}$

$473 - 86 = \underline{387}$

$131 - 88 = \underline{43}$

$342 - 92 = \underline{250}$

$285 - 96 = \underline{189}$

$514 - 185 = \underline{329}$

$620 - 191 = \underline{429}$

$708 - 289 = \underline{419}$

$856 - 298 = \underline{558}$

$997 - 398 = \underline{599}$

Four in a Flash

Effect: You show a sheet with a bunch of numbers (see below). You turn away and ask a spectator to cover up any 4 adjacent numbers using a little piece of paper you give him. You turn back, and ask the spectator to uncover the 4 numbers, and you attempt to add them in your head in less than 3 seconds. He uncovers them and you call out the total immediately. You both add up the numbers—your total is correct!

Props: Four in a Flash sheet and a Four in a Flash cover, which is just a 2"x1" piece of card stock big enough to cover up 4 numbers (2 in 1 row and 2 in the next row).

Secret: You see what number is 2 away diagonally from any of the 4 corners of the covering paper, and subtract that from 65. That is the total of the 4 covered numbers! For instance, in figure 2 below, the number 2 away diagonally from any of the 4 corners is 15. $65-15=50$, so 50 is the total of the 4 covered #s! Uncovering them bears this out: $1+7+18+24=50$.

Presentation:

“Did you know that I can add numbers very quickly in my head? Sometimes. We’ll see if today is one of those times!

When I turn around, I’d like you to put the small piece of paper so it covers any 4 of the numbers on this sheet. OK? (Turn around.) Let me know when you have covered the numbers. (Turn back, facing spectator & number sheet.)

(Note the number 2 away diagonally from 1 of the corners. As you talk, subtract it in your head from 65.) Now you could have covered any numbers. Any particular reason you covered those 4? (Note: This talk, and asking a question gives you a moment to subtract the # from 65 in your head.) When I say ‘lift’, I want you to lift the paper off the numbers. I will look at them quickly and attempt to add them in my head—in less than 5 seconds. Can you time me to see how long I take? Are you ready? (By now, you know the total. If you need more time, keep talking til you have it. Don’t have him lift the card til you have it.) Lift! (Immediately say the total when he lifts, doing it in just 1 second!) 50!

Let’s add them together. $1+7=8$. $+18 = 26$. $+24 = 50$. I’m glad you caught me on a good day!”

Tip: To help you remember to subtract the # 2 away diagonally from 65, on the Four in a Flash sheet, “65-2” appears in the lower right corner.

15	02	19	06	23	15	02
16	08	25	12	04	16	08
22	14	01	18	10	22	14
03	20	07	24	11	03	20
09	21	13	05	17	09	21
15	02	19	06	23	15	02
16	08	25	12	04	16	08

15	02	19	06	23	15	02
16	08	25	12	04	16	08
22	14			10	22	14
03	20			11	03	20
09	21	13	05	17	09	21
15	02	19	06	23	15	02
16	08	25	12	04	16	08

Subtraction Cards

Effect: You ask someone to think of a number 1-63 (Let the audience know what it is, but not you.) You hand him several cards that each have a bunch of numbers (see below). His number may be on 1 or several cards. He is to hand you all the cards that do NOT contain his number. You, after briefly looking at the cards he handed you, reveal his number!

Props: A set of 6 subtraction cards. (See below.)

1 3 5 7 9 11 13 15
17 19 21 23 25 27 29
31 33 35 37 39 41 43
45 47 49 51 53 55 57
59 61 63

2 3 6 7 10 11 14 15
18 19 22 23 26 27 30
31 34 35 38 39 42 43
46 47 50 51 54 55 58
59 62 63

4 5 6 7 12 13 14 15 20
21 22 23 28 29 30 31 36
37 38 39 44 45 46 47 52
53 54 55 60 61 62 63

8 9 10 11 12 13 14 15
24 25 26 27 28 29 30 31
40 41 42 43 44 45 46 47
56 57 58 59 60 61 62 63

16 17 18 19 20 21 22 23
24 25 26 27 28 29 30 31
48 49 50 51 52 53 54 55
56 57 58 59 60 61 62 63

32 33 34 35 36 37 38 39
40 41 42 43 44 45 46 47
48 49 50 51 52 53 54 55
56 57 58 59 60 61 62 63

Secret: Look at the first number of each card you are handed. Add them together. Then subtract the total from 63. That will be the thought of number. (Why 63? It is the total of the 6 first numbers.)

E.g. If you are handed the first two cards above, $1+2=3$. $63-3=60$, which is his number! If you are handed just the card starting with 4, his number would be $63-4=59$. If you are handed cards starting with 1,2,4,& 8, $1+2+4+8=15$. $63-15=48$, which would be his number.

Notes: For younger children, you can use less cards. E.g.

Cards 1-5	Think of # between 1-31	Subtract total from 31.
Cards 1-4	Think of # between 1-15	Subtract total from 15.

Math Learning: Subtracting numbers in your head.

12345679

Effect: You show a magic number: 12345679. Ask someone to name their favorite number 1-9. Suppose 6. Multiply the magic number $\times 6$. Then $\times 9$. What do you get? 666,666,666!

Props: blackboard & marker, or pencil and paper.

Secret/Teaching This: In the trick, we multiplied the magic number \times your number, then $\times 9$. What if we reverse that and first multiply it $\times 9$. What do you get? (Let students try.) Answer: 111,111,111. Hmm. So, no wonder whatever your number is 1-9, you wind up with all of that number, since $1 \times Y = Y$.

We are going to learn a lightning calculation trick for dividing by 9.

Presentation:

“You’ve heard of magic words. Abra-Kid-Abra, hocus pocus, presto change, etc. But have you ever heard of a magic number? I have one that I’ll write on the board: 12345679. Pretty magical, huh? You don’t seem very impressed, so let me show you why it’s magical. Can you name any number 1-9? (4) Multiply the magic number by 4. Now multiply what you got $\times 9$. What number do you get? 444,444,444! See, I told you it was a magic number!”

Rapid Division Tricks

Materials: For each child: Lightning Add/Subtract/Divide Worksheet, ~6 note cards, & pens or sharpees.

#1 Dividing by 5

What is $12 \div 5$? Answer: 2.4.

What is $134 \div 5$? Answer: 26.8

How do you arrive at these quickly?

With division, you can multiply both sides of the equation by the same number, and the answer is unchanged. Let's do this to make the problem easier by multiplying both sides by 2.

$12 \div 5 = 24 \div 10$. This is a much easier problem with the same answer. Just move the decimal on 24.0 1 space to the left to get 2.4

$$134 \div 5 = 268 \div 10 = 26.8$$

Try the practice problems on the worksheet.

#2 Dividing by 9 ($x \div 9$ where $x < 9$)

What is $2/9$? .2222

X divided by 9 (where $X < 9$) is X repeated.

For 2-digit numbers, the same is true if you divide by 99.

$$43/99 = ? \text{ .434343}$$

For 3 digit numbers, same divided by 999.

$$563 \div 999 = .563563$$

Instructor note: When the last digit > 5 , it gets rounded up at the end. e.g. $6 \div 9 = .66666\dots$ Rounded, this is .6667

Try the practice problems on the worksheet.

#3 Dividing by 9 ($x \div 9$ where $x > 9$)

Can you solve this problem: $9 \overline{)123}$

- Put the 1st digit in the dividend, in this case, 1, atop the line. $9 \overline{)123}$
- Add the 1 above the line to the next digit in the dividend, 2. $1+2=3$. Put 3 above the line. $9 \overline{)123}$
- Repeat. 3 (above the line) + 3 (next digit in dividend) = 6. Put 6 above the line. Since it's above the last digit in the dividend, it is the remainder. So the answer is: 13 remainder 6, or 13 $\frac{6}{9}$.

Another example:

$$9 \overline{)2222} = ? \quad 1) \begin{array}{r} 2 \\ 9 \overline{)2222} \end{array} \quad 2) \begin{array}{r} 24 \\ 9 \overline{)2222} \end{array} \quad 3) \begin{array}{r} 246 \\ 9 \overline{)2222} \end{array} \quad 4) \begin{array}{r} 246 \text{ r}8 \\ 9 \overline{)2222} \end{array} = 246 \frac{8}{9}$$

Try the practice problems on the worksheet.

Students Making Their Own Flash cards: Have students continue to build their lightning calc flashcard decks by writing, for each of the above problem types, 2-3 flash cards. Put a problem on 1 side, using 1 of the worksheet problems, or they can make up their own, and the answer on the back. Write large.

Human Calculator Division Worksheet

Part 1, Dividing by 5

$8 \div 5 = \underline{\quad}$ $14 \div 5 = \underline{\quad}$ $17 \div 5 = \underline{\quad}$ $22 \div 5 = \underline{\quad}$ $63 \div 5 = \underline{\quad}$

$86 \div 5 = \underline{\quad}$ $113 \div 5 = \underline{\quad}$ $231 \div 5 = \underline{\quad}$ $473 \div 5 = \underline{\quad}$ $834 \div 5 = \underline{\quad}$

Part 2, Dividing by 9 ($x \div 9$, $x < 9$)

$3 \div 9 = \underline{\quad}$ $4 \div 9 = \underline{\quad}$ $17 \div 99 = \underline{\quad}$ $25 \div 99 = \underline{\quad}$ $39 \div 99 = \underline{\quad}$

$48 \div 99 = \underline{\quad}$ $102 \div 999 = \underline{\quad}$ $281 \div 999 = \underline{\quad}$ $340 \div 999 = \underline{\quad}$ $476 \div 999 = \underline{\quad}$

Part 3, Dividing by 9 ($x \div 9$, $x > 9$)

$13 \div 9 = \underline{\quad}$ $35 \div 9 = \underline{\quad}$ $61 \div 9 = \underline{\quad}$ $152 \div 9 = \underline{\quad}$ $321 \div 9 = \underline{\quad}$

$403 \div 9 = \underline{\quad}$ $800 \div 9 = \underline{\quad}$ $1111 \div 9 = \underline{\quad}$ $3210 \div 9 = \underline{\quad}$ $4301 \div 9 = \underline{\quad}$



ANSWERS - Human Calculator Division Worksheet

Part 1, Dividing by 5

$8 \div 5 = \underline{1.6}$

$14 \div 5 = \underline{2.8}$

$17 \div 5 = \underline{3.4}$

$22 \div 5 = \underline{4.4}$

$63 \div 5 = \underline{12.6}$

$86 \div 5 = \underline{17.2}$

$113 \div 5 = \underline{22.6}$

$231 \div 5 = \underline{46.2}$

$473 \div 5 = \underline{94.6}$

$834 \div 5 = \underline{166.8}$

Part 2, Dividing by 9 ($x \div 9$, $x < 9$)

$3 \div 9 = \underline{.3333}$

$4 \div 9 = \underline{.4444}$

$17 \div 99 = \underline{.171717}$

$25 \div 99 = \underline{.252525}$

$39 \div 99 = \underline{.393939}$

$48 \div 99 = \underline{.484848}$

$102 \div 999 = \underline{.102102}$

$281 \div 999 = \underline{.281281}$

$340 \div 999 = \underline{.340340}$

$476 \div 999 = \underline{.476476}$

Part 3, Dividing by 9 ($x \div 9$, $x > 9$)

$13 \div 9 = \underline{1 \frac{4}{9}}$

$35 \div 9 = \underline{3 \frac{8}{9}}$

$61 \div 9 = \underline{6 \frac{7}{9}}$

$152 \div 9 = \underline{16 \frac{8}{9}}$

$321 \div 9 = \underline{35 \frac{6}{9}}$

$403 \div 9 = \underline{44 \frac{7}{9}}$

$800 \div 9 = \underline{88 \frac{8}{9}}$

$1111 \div 9 = \underline{123 \frac{4}{9}}$

$3210 \div 9 = \underline{356 \frac{6}{9}}$

$4301 \div 9 = \underline{477 \frac{8}{9}}$



Coins in the Envelope Prediction

Effect: You have a spectator think of a number and perform some numerical calculations, arriving at a final number. Whatever the final number is, the spectator should divide it by 2, and that, you state, is the amount of money in your envelope. E.g. if the spectator arrived at 10, you'll have 5 cents. If she has 8, you'll have 4 cents. She says that when she divides her number by 2, she gets 2 ½. You think she's joking, but she is not! You open your envelope and, sure enough, you have 2 pennies and half a penny!

Secret: The spectator always arrives at 5 which, divided by 2, is 2 ½.

Props: For each child: 2 pennies, a half penny, a 6.75 size white envelope, & a 2"x4" label with the below steps.

Mechanics:

1. Think of any number 1-10.
2. Double it.
3. Add 10.
4. Divide by 2.
5. Subtract the number you started with. (They will always be at 5 now.)
6. Divide by 2.

Preparation: Put the sticker on the envelope so you can remember the steps, if you need to refer to it. Put 2.5 cents inside the envelope.

Presentation:

"I have some money in my envelope. (Show envelope. You can read the steps from the envelope label.) I'm going to try a prediction. Can you think of a number 1-10, don't tell me what it is. Do you have one? (yes) Double it. Add 10. Divide it by 2. Subtract the number you started with. You are left with a final number, right? Whatever number you have, we'll divide it by 2 and that is how many cents I have in this envelope. In other words, if your final number is 10, I will have 5 cents. If it is 6, I will have 3 cents. Whatever number you are thinking of, divided by 2, I will have that many cents. When you divide your number by 2, what do you come up with? (2.5) Seriously, this is no time for jokes. The final number that you arrived at, divided by 2, what do you get? (2.5) 2.5? (yes) (Dump out the envelope contents, showing 2.5 cents.) You had me a little worried there for a moment!"

Notes:

- If performing for a group, have the spectator jot her calculations on a paper or blackboard, so everyone can follow along.
- Why does this work? Can they derive the algebra behind it and see why it works?
$$((2x+10)/2)-x = 5$$
$$(x+5-x) = 5.$$
$$5=5.$$
 The point: x can be anything and you still get 5.

What If...

If you add 14 instead of 10, what would the final number be? Answer: 7. However much you add, the final number is half of that. So if you have them add 14, you can end with 3.5 cents.

Math Learning: Multiplication, division, algebraic derivation, fractions.

Through the Brick Wall

Effect: Magician shows 2 puzzle pieces that fit together to form a solid brick wall. When they are turned to the other side and put together, a hole in the middle appears!

Props: Brick wall sheet; kids scissors

Art Project: Have each child cut their brick wall into 2 pieces, cutting along the subtle stepped line. The result is 2 stepped pieces that are the same size.

Secret: The 2 pieces can be put together in 2 ways. One shows a solid wall. The other shows a hole in the middle. To achieve the 2nd way, slide one piece down a step.

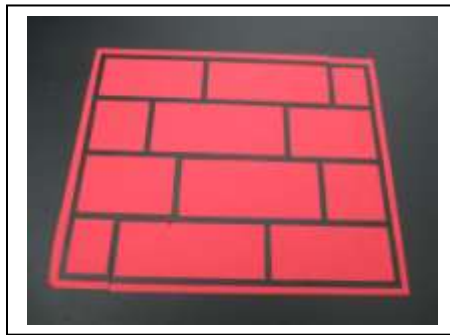
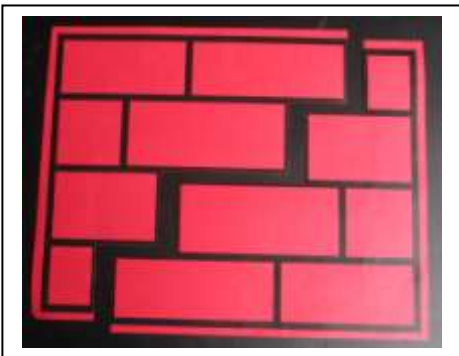
Presentation: You can talk about Harry Potter, and how when he went to the train station to go to Hogwarts, he went to track 9 $\frac{3}{4}$, but all there was was a solid brick wall. How did he get through it to get on the train?

The secret was to look on the other side of the wall. When he went to the other side (turn each piece over), he found a secret opening in the middle that he could slip through!

But if you're standing in front of the wall, you can't see it. (Turn the pieces back over, showing solid brick wall.)

Teaching Points:

- Have the kids hold the pieces up, rather than doing this on the table. It's more visible to an audience that way.



Giant Eraser

Effect: You need to erase something, and pull out a humongous eraser!

Props: Giant eraser.

Can you think of any ideas to have fun with this prop?



Calendar 9 (Pocket Puzzlers)

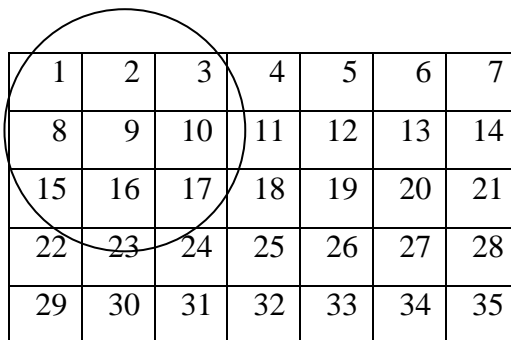
Effect: You show a calendar. While you look away, you ask a spectator to draw a circle around any 9 days in a 3x3 matrix. (See example below.) Then add the 9 dates on a calculator. You ask for the lowest number circled. Spectator gives it. You add the 9 numbers in your head instantaneously and announce the answer, which proves to be correct!

Props: Calendar pad and pencil.

Secret: You add 8 to the number the spectator gives you, which tells you the middle number (i.e. the average number) of the 9. You multiply it by 9, using the 9s multiplication trick previously learned.

Presentation: “Ever wish there was more time in the day? I have solved that problem. On this calendar, I’ve added a few extra days to the end of the month! I’m going to look away. When I do, I’d like you to draw a circle around any 9 days in a 3x3 configuration. For instance, you might draw a circle here or here (without making a mark on the page, move the pencil in a circle around 9 dates to be sure he knows what you mean) – wherever you like. Got it?”

(Look away as spectator draws a circle around 9 dates in 3x3 configuration.) Let me know when you have done that. (Spectator says he has.) Good. Show your dates to the audience. Now, I’d like you to add the 9 dates on the calculator. Let me know when you have the total. Would you jot it on the board. (where you cannot see it) Tell me the lowest number of the dates you circled. I am going to see if I can add the 9 dates in my head... Is the total you came up with... ___? (yes) A round of applause for my assistant, please.”



1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35

Calendar Magic (Pocket Puzzlers)

Note: This is a series of 3 tricks using a calendar page. 1) Demonstrate a trick. 2) Teach it. See if the students can figure it out. 3) Let students practice. Then repeat this sequence with the other 2. At the end, optionally have 3 students come up, each demonstrating 1 of the calendar tricks to the whole group. Props are the same for each trick: a pad of calendar pages for each student, and a pencil. Note: The calendar has 35 days to provide more selection possibilities.

Trick #1: Favorite Day

Effect: You show a calendar page (see below). While you look away, ask a spectator to think of her favorite day and draw a circle encompassing 3 dates on that day. (See example.) The spectator tells you the total of the dates (51). You, then, immediately name the 3 dates (10,17, & 24)!

Sun	Mon	Tue	Wed	Thu	Fri	Sat
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35

Secret: Divide the spectator's total by 3. This tells the middle number of the 3 ($51/3=17$ in this example). How, then, do you get the 3 dates? Subtract 7 to get the low number of the 3. Then add 7, and 7 again to get the other 2 dates!

Trick #2: Favorite Week

Effect: This time while you look away, you ask a spectator to think of their favorite week in this calendar, and draw a circle encompassing 5 days in that week. (See example.) The spectator tells you the total of the dates (55). You, then, immediately name the 5 dates (9,10,11, 12, & 13)!

Sun	Mon	Tue	Wed	Thu	Fri	Sat
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35

Secret: How do you do this one? Since 5 dates were chosen, divide the total by 5 to get the average, or middle number in this case. ($55/5=? 11$) How do you then know the 5 dates? The average is the middle, or 3rd of the 5 dates. Subtract 2 to get the starting date. Name it and the next 4 (9,10,11,12,13).

Trick #3: Favorite Square

Effect: In this last version, while you look away, ask a spectator to Draw a square around any 4 dates (See example.) The spectator tells you the total of the dates (92). You, then, immediately name the 4 dates (19,20,26, & 27)!

Sun	Mon	Tue	Wed	Thu	Fri	Sat
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35

Secret: This one is harder. But doable with practice. How many dates were chosen? 4. So what do you divide the total by? 4. $92/4=? 23$. (You can use pencil and paper on this one if you wish.) How do you get from 23 to the 4 dates? 23 is the average, but it is not any of the dates. What do you do? Subtract 4 to get the first date. $23-4=19$. The next date is 1 higher, 20. How do you get the last 2 dates? Add 7 to each. $19+7=26$. So 26 & 27.

T-Shirt Card Trick (Art Project)

Effect: Spectator selects a card. Amidst clever patter (see presentation), magician reveals the card on the label inside his collar!

Props: T-shirt with rabbit in hat drawing; box of fabric-marking crayons; card deck; Black marker or printed sticker that says “2♣”; ¼ page sheet with instructions for washing the shirt.

Secret: Although it seems as though the spectator has a free choice, you actually force him to choose the card on your label, using the cross cut force. This is explained in the glasses trick instructions.



Preparation:

1. Have the kids color in their shirts using the fabric crayons. Tell the kids when they go home, to have mom iron the shirt, permanently sealing in the colors. Give them the ¼ page sheet that has instructions for mom.
2. Write 2♣ in black marker on the shirt label (on the collar). Or, put a 2♣ sticker over the label. Whichever works best.
3. Have the 2♣ secretly on top of the (face down) deck.

Presentation: “I need a volunteer from the audience. Thank you. Would you cut the cards approximately in half. I’ll mark your cut.

(Divert attention for a moment.) Incidentally, do you play card very much? What sort of cards do you play? Very interesting. Take the card that you cut to (lift off top half, handing him the force card—supposedly the “free choice” he cut to), show it around, and don’t let me see it. Did everyone see it? Put it back in the middle of the deck.

I am going to see if I can guess the name of your card. In fact, I’m so sure I can that if I’m wrong, I’ll give you, ah—well, how about the shirt off my back! I will look through the cards (hold them between your hands, spread them faces toward you, & pull 1 out—not his!), and here is your card, the 8 of hearts! Right? (no) Oh. Well, sometimes it takes me 2 tries. The 7 of spades! (No) What was your card? (2 of clubs) Well, I guess I have to give you my shirt. Sorry it’s not cleaner; though it is 100% cotton—I think. Can you read what it says on the label? (Pull out the label. Have him read out “2 of clubs”!)

Placard Number Prediction (extra)

Effect: Explain that you'll jot down a prediction. Then you'll ask a spectator for a 3-digit number. You will then reverse it and subtract the two, then reverse the total and add, coming up with a final number which, hopefully, will match your prediction. You do all this, show your prediction at the end—but but it's wrong! But wait a minute, you turn it upside down. It is correct after all!

Props: pencil & 2 sheets of paper

Secret: The number always comes out to 1089!

Preparation: Write 6801 on an 8.5 x 11 sheet of paper. Put a curl in the 6 so it'll look like a 9 when upside down. Put a pencil dot on the back of the sheet at the top, so you know that when you lift up the page & the dot is up, it'll say "6801".

Mechanics:

1. _____ Spectator names any 3 digit number. All digits are different. E.g. 752
2. _____ Reverse the number. E.g. 257 (If reversed # is larger than first #, put reversed # on top.)
3. _____ Subtract. E.g. 495
4. _____ Reverse the number. E.g. 594
5. _____ Add the last 2 numbers. E.g. 1089

Presentation:

"I'm feeling lucky today so I've made a prediction (show back of the paper & set it down), which we'll get to a little later. I'm going to ask someone to call out a 3 digit number—any 3 digit number they want. I will then reverse it and subtract, then reverse the total and add, coming up with a final total. With any luck, it will match my prediction. We'll see if luck is with me today. Can someone call out any 3 digit number where the digits are all different. (E.g. 752.) Uh oh. No one has ever called out that one before. Are you sure you don't want to change your mind? (Jot it on the board.) I will reverse it. (Do so. If reversed number is larger, put it on top. If it's smaller, put it beneath.) Now I'll subtract. (Talk out loud as you do.) Please check my arithmetic.

Now I will reverse the total (do so, putting it beneath). And finally, I'll add these last 2 numbers together. Now you could have selected any number. We wound up with 1089. You remember that before we started, I made a prediction. Believe it or not, I predicted...6801? Hmm. Wait a minute. What was the total we wound up with? (1089) Oh, I'm sorry, I had my prediction upside down. (Turn it upside down, showing 1089!) You had me a little worried there for a moment!"

Notes:

- The pencil dot on the back of the prediction is important. If you pick up the prediction the wrong way and it says 1089, you'll lose the gag in the trick—not as effective.
- If you get 99 in step 3 above, tack a 0 onto the front. 099. When you reverse, you get 990. $990+099=1089$.
- If you have a small audience and write on a paper, make sure it is positioned so both you and your audience can read the numbers. I.e. don't have it upside down to either of you.

Math Learning: Addition and subtraction of 3 digit numbers.

The Psychic Mathematician (extra)

Effect: You have been struck by a premonition, so you jot a prediction and set it aside on the table. 1-3 spectators create 3 different 3 digit numbers as follows: For each 3-digit number, the first number is 1,2, or 3. The 2nd is 4,5,or 6. And the 3rd digit is 7, 8, or 9. A digit can only be used once. So, e.g., the 3 #s might be:

159
368
247

You add these and get...774. Which matches your prediction!

Props: pencil and paper.

Secret: The answer is always 774. Ask the students why. The last column is always going to have a 7, an 8, and a 9. Does the order matter? No. Same with the other 2 columns.

Logistics: If you have 3 spectators, have each choose 1 of their numbers to help create each 3-digit number. If you just have 1 spectator, he can create all the digits, staying within your guidelines.

Extra No Prop Math Pocket Puzzlers

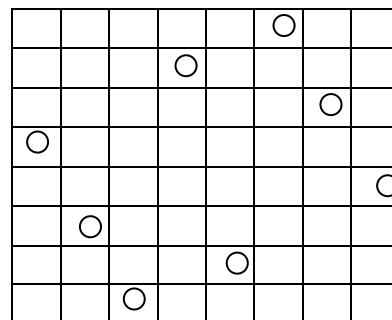
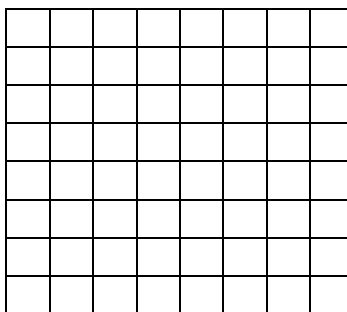
8 Queens

Draw an 8x8 grid (see diagram # 1).

Challenge: Can you place 8 pennies (or other small objects—bits of paper or whatever) on the grid such that no 2 are in the same row, column, or diagonal? Each object occupies a different square.

Props: Pencil and paper.

Note: There are multiple answers. One is pictured here.



How Many Sides?

Activity: From an 8.5x11 paper, cut a strip ~1”x11”. Bend it into a loop, make 1 twist, and tape it, forming a ring with 1 twist in it. Ask how many sides your loop of paper has. Paper has 2 sides, right? But the correct answer with this piece is 1! How is that? Draw a line, going all the way around the loop. You’ll see that (because of the single twist), it makes 1 single line, ending where it starts! Which shows that you have, indeed, a 1-sided piece of paper! This is called a Mobius Strip, named after the mathematician who discovered it.

Props: Paper, tape, pen or pencil.

Calculating the Answer

Claim that you have such great mental math abilities that your friend can give you any 2 numbers, you will multiply them in your mind, without using any other items, and tell him the correct answer. Does he think you can do that? Your friend thinks of 2 numbers and multiplies them—either on paper or using a calculator. He sees the answer. He tells you the numbers. E.g. 462 x 597. You concentrate briefly and say “the correct answer”. This is a play on words. You don’t solve the problem. You said you’d tell him “the correct answer”!

Note: You can also do this with adding (or any operation). Use several #s if you’d like. E.g. 1976 + 586 + 93

Props: None.

Can you arrange 8 4’s so they total 176?

You can put them together in any configuration: e.g. 444, 4444, etc. You can also provide a hint if you wish: This just uses adding. Answer: $44+44+44+44 = 176!$

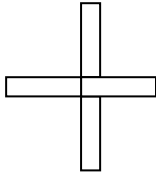

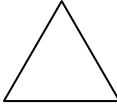
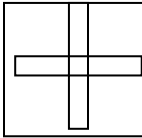
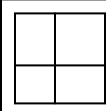
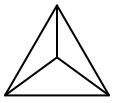
Props: Paper & pencil.


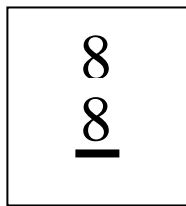
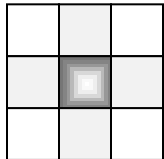
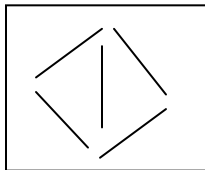

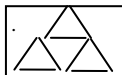
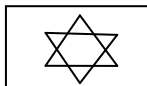
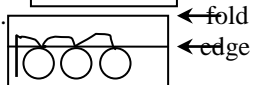
Can you arrange 5 6’s so they total 19?

Answer: $6+6+6+6/6 = 19!$

Props: pencil & paper.

Mathemagic Puzzles

<p>1. Apples</p> <p>If you take 6 apples from 8 apples, how many apples do you have?</p> <p style="text-align: right; font-size: small;">www.abrakid.com © 2012 0023s</p>	<p>2. 26 Cents</p> <p>I have 2 American coins in my hand that total 26 cents. One of them is not a penny. What are they?</p> <p style="text-align: right; font-size: small;">www.abrakid.com © 2012 0055r</p>	<p>3. All Squared Up</p> <p>Put 4 crayons in a plus sign, as in the below diagram.</p> <p>Challenge: can you make a square, moving just 1 crayon?</p> <div style="text-align: center;">  </div> <p style="text-align: right; font-size: small;">www.abrakid.com © 2012 0008g</p>	<p>4. An Equalizing Move</p> <p>You have 2 cards. #1 says: $1+9$. #2 says: $3+4$. In 1 move, can you make the answers to the 2 problems be equal? You cannot add, cross out, or take away numbers. Work with what you have.</p> <p style="text-align: right; font-size: small;">www.abrakid.com © 2012 0024a</p>
<p>5. Sheep in Pens</p> <p><u>Sheeps & Pens:</u> Farmer Bob has 9 sheep & 4 pens. The Agriculture Department, after visiting his farm, recommends that Farmer Bob put an odd number of sheep in each of the 4 pens. Can you show Bob how he might do this.</p> <p style="text-align: right; font-size: small;">www.abrakid.com © 2012 0014e</p>	<p>6. A Rat in the House</p> <p>A Rat in the House Might Eat the Ice Cream.</p> <p>What do the first letters of these words spell?</p> <p style="text-align: right; font-size: small;">www.abrakid.com © 2012 0045?</p>	<p>7. Shape Up!</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> </div> <p>Can you divide a square into 4 equal squares?</p> <p>Can you divide a triangle into 3 equal triangles?</p> <p style="text-align: right; font-size: small;">www.abrakid.com © 2012 0015g</p>	<p>8. Circumference Vs. Height</p> <p>What's bigger, you ask a spectator, the circumference of the cup or the height of the cup? Put something under the cup to raise the height from the table. Ask again. Keep raising height & asking until spectator says the height is bigger. Pull out a string or paper & show that circumference is bigger.</p> <p style="text-align: right; font-size: small;">www.abrakid.com © 2012 0037g</p>
<p>9. Riddle of the Sphinx</p> <p>In Greek mythology, the Sphinx guarded the gates of the city of Thebes with this riddle:</p> <p>What goes on 4 legs in the morning, on 2 legs at noon, and on 3 legs at dusk?</p> <p>Can you solve it?</p> <p style="text-align: right; font-size: small;">www.abrakid.com © 2012 0051r</p>	<p>10. Easy as 1-2-3</p> <p>Set out 2 pennies, 2 nickels, & 2 dimes.</p> <p>Challenge: Using only these 6 coins, can you put them in a row so there is 1 coin between the pennies, 2 coins between the nickels, & 3 coins between the dimes?</p> <p style="text-align: right; font-size: small;">www.abrakid.com © 2012 0033r</p>	<p style="text-align: center;">Solutions 1A</p> <ol style="list-style-type: none"> 1. 6. 2. A quarter & a penny. One isn't a penny—the other is! 3. <div style="display: flex; align-items: center;">  <div style="margin-left: 10px;">Move 1 outward, making a sq in the middle.</div> </div> 4. Turn $1+9$ upsidedown. 5. Put 1 sheep in each of 3 pens, 6 in the 4th. Put 1 of the pens inside the 4th. <p style="text-align: right; font-size: small;">www.abrakid.com © 2012</p>	<p style="text-align: center;">Solutions 1B</p> <ol style="list-style-type: none"> 6. arithmetic 7. <div style="display: flex; align-items: center;">   </div> 8. Beforehand measure both with a string, to see how high you can raise it & still have circumference be greater. 9. A person (crawls as a baby on all 4's, walks in middle, has a cane late.) 10. D P N P D N <p style="text-align: right; font-size: small;">www.abrakid.com © 2012</p>

<p>11. Half of 8 = 3?</p> <p>My teacher says that half of 8 is 3. How can this be? Or has he lost his marbles?</p> <p>www.abrakid.com © 2012 0001c</p>	<p>12. Time Warp</p> <p>When I wear my green shirt, it takes me 1 minute and 20 seconds to write the alphabet.</p> <p>However, when I wear my purple shirt, it takes me 80 seconds to write the alphabet.</p> <p>How do you explain this?</p> <p>www.abrakid.com © 2012 0049r</p>	<p>13. 5 Triangles</p>  <p>With 9 toothpicks, form 3 equilateral triangles as above.</p> <p>The challenge: Move 3 toothpicks to make 5 equilateral triangles. (You cannot bend or break them.)</p> <p>www.abrakid.com © 2012 0029g</p>	<p>14. 8 Triangles, 6 Toothpicks?</p> <p>Can you make 8 triangles using only 6 toothpicks (or straws)?</p> <p>No bending or breaking.</p> <p>www.abrakid.com © 2012 0038g</p>
<p>15. Writing 1000</p> <p>Can you write “1000” without the pen leaving the paper?</p> <p>No marks should be visible connecting the numbers. And the pen must make a mark whenever it moves across the paper.</p> <p>www.abrakid.com © 2012 0006c</p>	<p>16. Days of the Month</p> <p>Some months have 30 days. Some have 31. Which months have 28 days?</p> <p>www.abrakid.com © 2012 0019r</p>	<p>17. Bus Trip</p> <p>You're the bus driver. To begin, your bus has 0 kids. Your 1st stop you pick up 3 kids. 2nd stop: pick up 3 more, 1 gets off. 3rd stop: 3 get off, 1 gets on.</p> <p>What color is the driver's hair?</p> <p>www.abrakid.com © 2012 0010s</p>	<p>18. 8+8=10000?</p>  <p>Can you, in 1 move, make this = 1000?</p> <p>www.abrakid.com © 2012 0052a</p>
<p>19. 9 Boxes</p>  <p>Can you put numbers in the 8 outside boxes above so the top & bottom row, and the left & right column all total 9? White squares must each get the same #, and gray squares must also each get the same #. Can you come up with more than 1 solution?</p> <p>www.abrakid.com © 2012 0012a</p>	<p>20. Making a Diamond</p>  <p>Use 5 toothpicks to make a rhombus (diamond) with a line in the middle as above. Challenge: Can you take away 3 toothpicks, put back 2, and be left with the same diagram?</p> <p>www.abrakid.com © 2012 0058g</p>	<p>Solutions 2A</p> <p>11. Draw a line thru the 8. The right half is a 3! </p> <p>12. They are the same amounts of time.</p> <p>13. </p> <p>14. </p> <p>15. </p> <p>Fold the paper. Draw “1000”. Connecting lines go above edge.. Unfold and just “1000” shows. Connecting lines are on back.</p> <p>www.abrakid.com © 2012</p>	<p>Solutions 2B</p> <p>16. All of them!</p> <p>17. Whatever color your hair is, since you are the bus driver.</p> <p>18. Turn it sideways.(10000).</p> <p>19. white,gray: 1,7 or 2,5 or 3,3 or 4,1.</p> <p>20. Move 3 to the side. Then move the last 2 over to the 3, reconstructing the same shape!</p> <p>www.abrakid.com © 2012</p>